Learning Based Partial Differential Equations for Visual Processing

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Outline

- Background and Motivation
- Learning Based PDE
- Applications
- Conclusions

What is (Evolutionary) PDE?

- Heat Equation

$$\begin{cases} \frac{\partial I}{\partial t} &= \Delta I, \\ I\big|_{t=0} &= I_0, \\ \frac{\partial I}{\partial n}\big|_{\partial D} &= 0. \end{cases}$$

Governing Eqn.

Initial Condition

Boundary Condition

A.K. Jain. Partial differential equations and finite-difference methods in image processing, part 1. *Journal of Optimization Theory and Applications*, 23:65–91, 1977.

How to Use PDE for Image Proc.?

- Heat Equation

$$\begin{cases} \frac{\partial I}{\partial t} & = & \Delta I, \\ I\big|_{t=0} & = & I_0, \\ \frac{\partial I}{\partial n}\big|_{\partial D} & = & 0. \end{cases}$$

Input Image

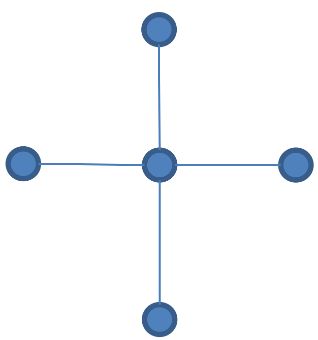
- Discretization

$$\frac{\partial I}{\partial t} \approx \frac{I(x,y,t+\Delta t) - I(x,y,t)}{\Delta t}$$

Explicit Scheme

$$\frac{\partial I}{\partial t} \approx \frac{I(x,y,t) - I(x,y,t-\Delta t)}{\Delta t}$$

Implicit Scheme



$$\frac{\partial I(x,y,t)}{\partial x} \approx \frac{I(x+\Delta x,y,t) - I(x,y,t)}{\Delta x}$$

$$\frac{\partial^2 I(x,y,t)}{\partial x^2} \approx \frac{I(x+\Delta x,y,t) - 2I(x,y,t) + I(x-\Delta x,y,t)}{(\Delta x)^2}$$

A.K. Jain. Partial differential equations and finite-difference methods in image processing, part 1. *Journal of Optimization Theory and Applications*, 23:65–91, 1977.

How to Use PDE for Image Proc.?

- Heat Equation

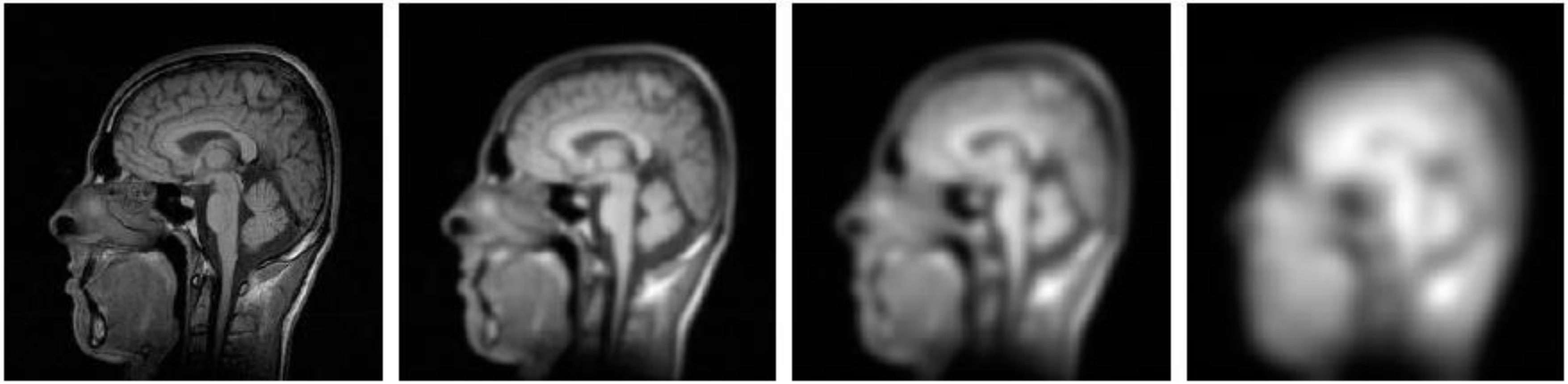
$$\begin{cases} \frac{\partial I}{\partial t} & = & \Delta I, \\ I\big|_{t=0} & = & I_0, \\ \frac{\partial I}{\partial n}\big|_{\partial D} & = & 0. \end{cases}$$

Input Image

- Discretization

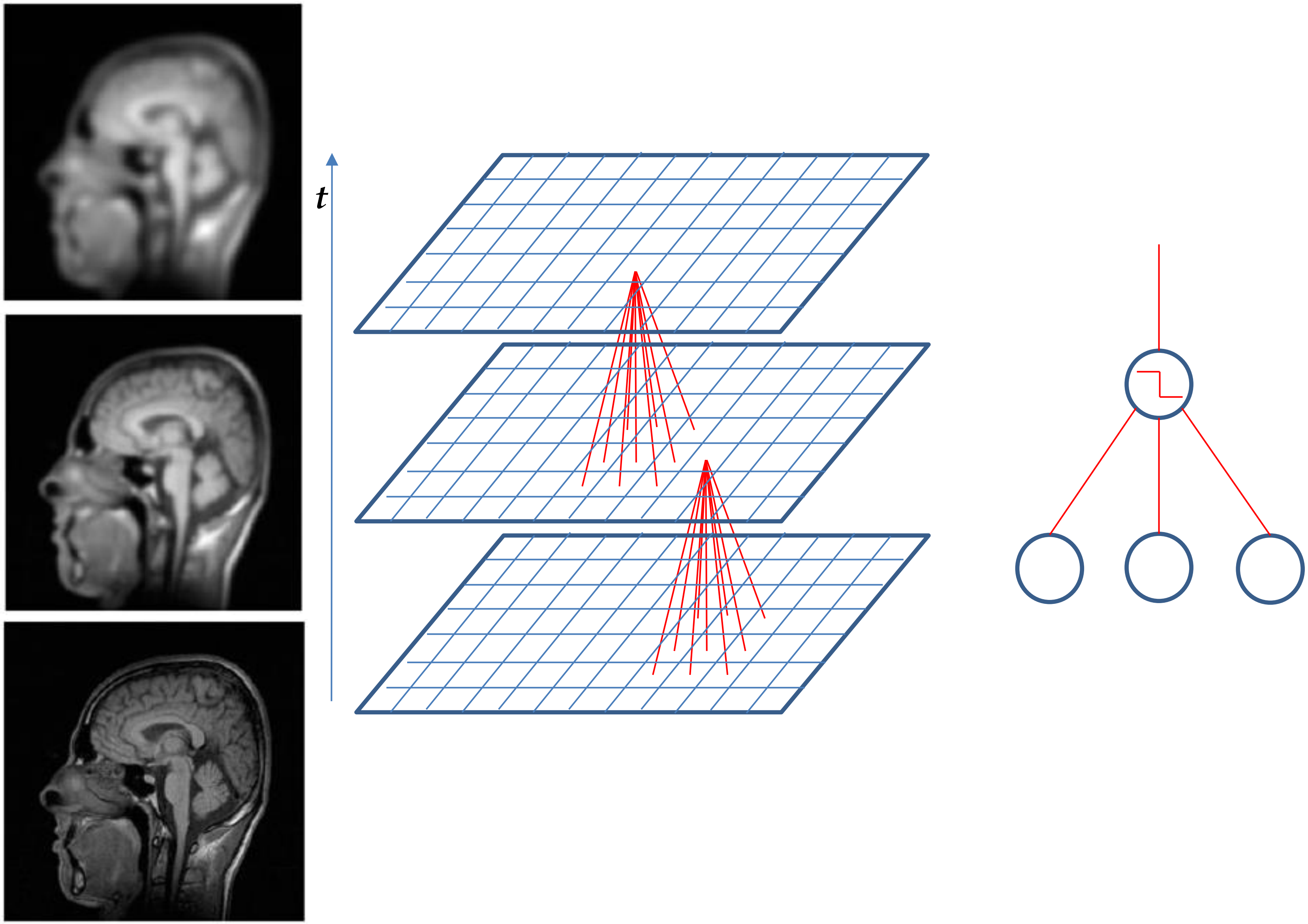
$$\frac{\partial I}{\partial t} = \Delta I \quad \Rightarrow$$
$$\frac{I(x,y,t+\Delta t) - I(x,y,t)}{\Delta t} =$$
$$(I(x+\Delta x, y, t) + I(x-\Delta x, y, t) + I(x, y+\Delta y, t) + I(x, y-\Delta y, t))/4.$$

Explicit Scheme



A.K. Jain. Partial differential equations and finite-difference methods in image processing, part 1. *Journal of Optimization Theory and Applications*, 23:65–91, 1977.

Connection to Neural Networks



A Brief History of PDE Methods

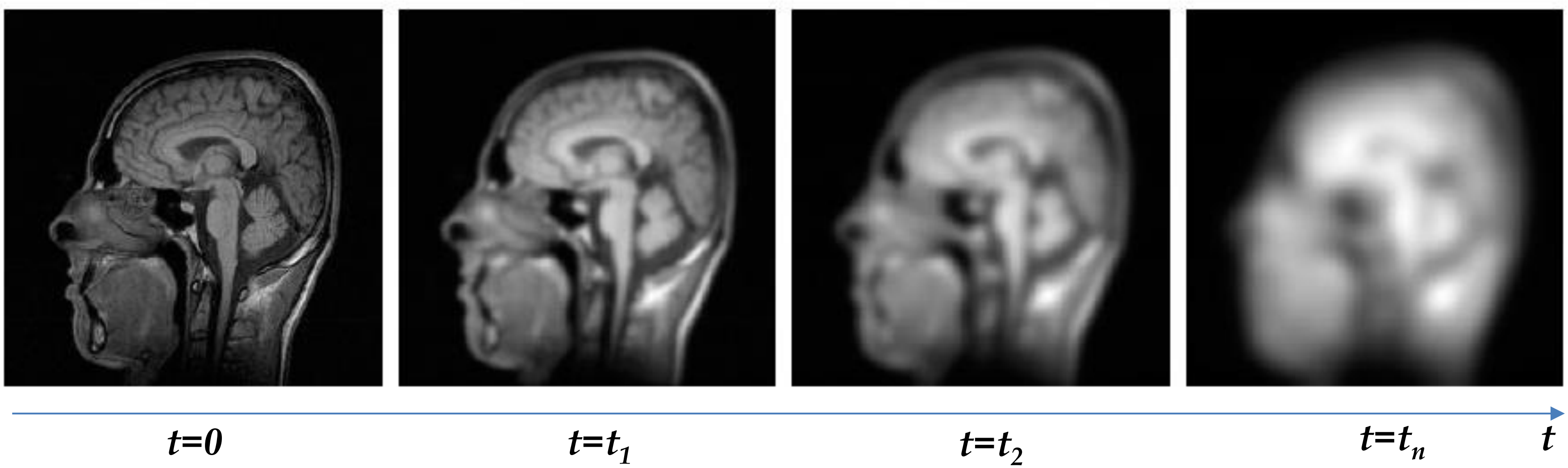
- Scale Space

$$I_\sigma = I_0 * G(\sigma^2, \mathbf{x}).$$

- Heat Equation

$$\left\{ \begin{array}{rcl} \frac{\partial I}{\partial t} & = & \Delta I, \\ I\big|_{t=0} & = & I_0, \\ \frac{\partial I}{\partial n}\big|_{\partial D} & = & 0. \end{array} \right.$$

$$\boxed{t = \frac{1}{2}\sigma^2}$$



A. Witkin. Scale-space filtering. In *Proc. Int. Joint Conf. Artificial Intelligence*, 1983.
J. Koenderink. The structure of images. *Biological Cybernetics*, 50:363–370, 1984.

A Brief History of PDE Methods

- Anisotropic PDEs

$$\begin{cases} \frac{\partial I}{\partial t} &= \nabla \cdot (c(\|\nabla I\|)\nabla I), \\ I\big|_{t=0} &= I_0, \\ \frac{\partial I}{\partial n}\big|_{\partial D} &= 0. \end{cases}$$

$$c(x) = \frac{1}{1+(x/K)^2} \text{ or } \exp(-(x/K)^2).$$

P. Perona and J. Malik. Scale-space and edge detection using anisotropic diffusion. *IEEE TPAMI*, 12(7):629–639, 1990.

A Brief History of PDE Methods

- Shock Filters

$$\begin{cases} \frac{\partial I}{\partial t} & = & -\|\nabla I\| F(\Theta(I)), \\ I\big|_{t=0} & = & I_0, \\ \frac{\partial I}{\partial n}\big|_{\partial D} & = & 0. \end{cases}$$

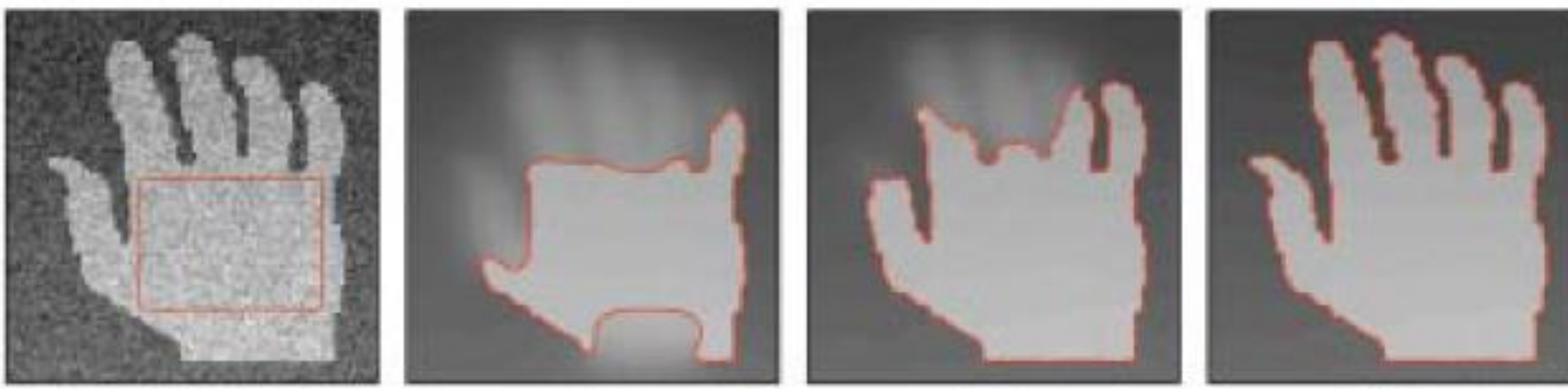
S.J. Osher and L. I. Rudin. Feature-oriented image enhancement using shock filters. *SIAM J. Numerical Analysis*, 27(4):919–940, 1990.

A Brief History of PDE Methods

- Active Contours

$$\min_{C} \int_0^1 g(\|\nabla I(C(p))\|^2 \|C'(p)\| \, \mathrm{d}p.$$

Variational Calculus → Euler-Lagrange Equation → Gradient Descent → Evolutionary PDEs



G. Aubert and P. Kornprobst. *Mathematical Problems in Image Processing*. Springer-Verlag, 2002.

Summary

- Two kinds of approaches
  - Direct design: write down PDEs directly
  - Variational design: energy functional → Euler-Lagrange equation
- Existing applications of PDEs
  - Denoising
  - Enhancement
  - Segmentation
  - Stereo
  - Inpainting
  - …

- It was as hot as artificial neural network in 1990s!

G. Aubert and P. Kornprobst. *Mathematical Problems in Image Processing*. Springer-Verlag, 2002.

But…

- Designing PDEs is too difficult!
  - High math skills
  - Good insights into the problem

- Can we have a convenient way?

Possible!

PDEs + Learning = Learning Based PDEs

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Basic Idea

- Observe the invariant properties of vision problems
- Determine differential invariants
- Determine combination coefficients among invariants
  - By PDE-constrained optimal control

- A user only have to prepare input/output training data!

- The SAME framework for various problems

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

General PDEs

$$
\left\{
\begin{array}{rcll}
f_t & = & L(\langle u \rangle, \langle f \rangle), & (\mathbf{x}, t) \in Q, \\
f & = & 0, & (\mathbf{x}, t) \in \Gamma, \\
f\big|_{t=0} & = & f_0, & \mathbf{x} \in \Omega.
\end{array}
\right.
$$

$\mathbf{x}$	(x, y), spatial variable	t	temporal variable
Ω	an open region of R^2	$\partial\Omega$	boundary of Ω
Q	$\Omega \times (0, T)$	Γ	$\partial\Omega \times (0, T)$
∇f	gradient of f	$\mathbf{H}_f$	Hessian of f
$\wp$	$\{\emptyset, x, y, xx, xy, yy, \cdots\}$	$\|p\|$, $p \in \wp \cup \{t\}$	the length of string p
$\dfrac{\partial^{\|p\|} f}{\partial p}$, $p \in \wp \cup \{t\}$	$f, \dfrac{\partial f}{\partial t}, \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial^2 f}{\partial x^2}, \dfrac{\partial^2 f}{\partial x \partial y}, \cdots$, when $p = \emptyset, t, x, y, xx, xy, \cdots$		
f_p, $p \in \wp \cup \{t\}$	$\dfrac{\partial^{\|p\|} f}{\partial p}$	$\langle f \rangle$	$\{f_p \| p \in \wp\}$
$L_{\langle f \rangle}(\langle f \rangle, \cdots)$	the differential operator $\sum_{p \in \wp} \dfrac{\partial L}{\partial f_p} \dfrac{\partial^{\|p\|}}{\partial p}$ associated to function $L(\langle f \rangle, \cdots)$		

Our PDEs

$$
\begin{cases}
O_t = L_O(\mathbf{a}, \langle O \rangle, \langle \rho \rangle), & (\mathbf{x}, t) \in Q, \\
O = 0, & (\mathbf{x}, t) \in \Gamma, \\
O|_{t=0} = O_0, & \mathbf{x} \in \Omega; \\
\rho_t = L_\rho(\mathbf{b}, \langle \rho \rangle, \langle O \rangle), & (\mathbf{x}, t) \in Q, \\
\rho = 0, & (\mathbf{x}, t) \in \Gamma, \\
\rho|_{t=0} = \rho_0, & \mathbf{x} \in \Omega.
\end{cases}
$$

ρ: indicator function, for collecting large scale information.

$\mathbf{a} = \{a_i\}$ and $\mathbf{b} = \{b_i\}$ are control functions.

$\mathbf{x}$	(x, y), spatial variable	t	temporal variable
Ω	an open region of R^2	$\partial\Omega$	boundary of Ω
Q	$\Omega \times (0, T)$	Γ	$\partial\Omega \times (0, T)$
$\langle f \rangle$	all the spatial derivatives of f		

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Two Basic Invariances

- Shift Invariance
- Rotation Invariance

Theorem 1: Coefficients $\{a_j\}$ and $\{b_j\}$ must be independent of $\mathbf{x}$.

Theorem 2: L_O and L_ρ must be functions of **fundamental differential invariants** that are invariant under shift and rotation.

Fundamental differential invariants can be viewed as "**bases**" of PDEs.

P. Olver. *Applications of Lie Groups to Differential Equations, Springer-Verlarg*. 1993.

Shift/Rotation Invariant Fundamental Differential Invariants

Table 1: Shift and rotationally invariant fundamental differential invariants up to second order.

i	$\mathrm{inv}_i(\rho, O)$
0,1,2	$1,\ \rho,\ O$
3,4,5	$\|\|\nabla\rho\|\|^2 = \rho_x^2 + \rho_y^2,\ (\nabla\rho)^t\nabla O = \rho_x O_x + \rho_y O_y,\ \|\|\nabla O\|\|^2 = O_x^2 + O_y^2$
6,7	$\mathrm{tr}(\mathbf{H}_\rho) = \rho_{xx} + \rho_{yy},\ \mathrm{tr}(\mathbf{H}_O) = O_{xx} + O_{yy}$
8	$(\nabla\rho)^t\mathbf{H}_\rho\nabla\rho = \rho_x^2\rho_{xx}^2 + 2\rho_x\rho_y\rho_{xy}^2 + \rho_y^2\rho_{yy}^2$
9	$(\nabla\rho)^t\mathbf{H}_O\nabla\rho = \rho_x^2 O_{xx}^2 + 2\rho_x\rho_y O_{xy}^2 + \rho_y^2 O_{yy}^2$
10	$(\nabla\rho)^t\mathbf{H}_\rho\nabla O = \rho_x O_x\rho_{xx} + (\rho_y O_x + \rho_x O_y)\rho_{xy} + \rho_y O_y\rho_{yy}$
11	$(\nabla\rho)^t\mathbf{H}_O\nabla O = \rho_x O_x O_{xx} + (\rho_y O_x + \rho_x O_y)O_{xy} + \rho_y O_y O_{yy}$
12	$(\nabla O)^t\mathbf{H}_\rho\nabla O = O_x^2\rho_{xx} + 2O_x O_y\rho_{xy} + O_y^2\rho_{yy}$
13	$(\nabla O)^t\mathbf{H}_O\nabla O = O_x^2 O_{xx} + 2O_x O_y O_{xy} + O_y^2 O_{yy}$
14	$\mathrm{tr}(\mathbf{H}_\rho^2) = \rho_{xx}^2 + 2\rho_{xy}^2 + \rho_{yy}^2$
15	$\mathrm{tr}(\mathbf{H}_\rho\mathbf{H}_O) = \rho_{xx}O_{xx} + 2\rho_{xy}O_{xy} + \rho_{yy}O_{yy}$
16	$\mathrm{tr}(\mathbf{H}_O^2) = O_{xx}^2 + 2O_{xy}^2 + O_{yy}^2$

P. Olver. *Applications of Lie Groups to Differential Equations, Springer-Verlarg*. 1993.

Simplest PDEs

$$
\begin{aligned}
L_O(\mathbf{a}, \langle O \rangle, \langle \rho \rangle) &= \sum_{j=0}^{16} a_j(t)\mathrm{inv}_j(\rho, O), \\
L_\rho(\mathbf{b}, \langle \rho \rangle, \langle O \rangle) &= \sum_{j=0}^{16} b_j(t)\mathrm{inv}_j(O, \rho).
\end{aligned}
$$

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Learning Coefficients by Optimal Control

$$\min J\left(\{O_m\}_{m=1}^{M}, \{a_j\}_{j=0}^{16}, \{b_j\}_{j=0}^{16}\right)$$
$$= \frac{1}{2}\sum_{m=1}^{M}\int_{\Omega}[O_m(\mathbf{x},1) - \tilde{O}_m(\mathbf{x})]^2 \mathrm{d}\Omega + \frac{1}{2}\sum_{j=0}^{16}\lambda_j \int_0^1 a_j^2(t)\mathrm{d}t + \frac{1}{2}\sum_{j=0}^{16}\mu_j \int_0^1 b_j^2(t)\mathrm{d}t,$$

$$\left\{\begin{array}{rcll}
O_t & = & \sum\limits_{j=0}^{16} a_j(t)\mathrm{inv}_j(\rho, O), & (\mathbf{x}, t) \in Q, \\
O & = & 0, & (\mathbf{x}, t) \in \Gamma, \\
O|_{t=0} & = & O_0, & \mathbf{x} \in \Omega; \\
\rho_t & = & \sum\limits_{j=0}^{16} b_j(t)\mathrm{inv}_j(O, \rho), & (\mathbf{x}, t) \in Q, \\
\rho & = & 0, & (\mathbf{x}, t) \in \Gamma, \\
\rho|_{t=0} & = & \rho_0, & \mathbf{x} \in \Omega.
\end{array}\right.$$

$(I_m, \tilde{O}_m)$ are training samples, where I_m is the input image and $\tilde{O}_m$ is the expected output image, $m = 1, 2, \cdots, M$.

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Solving Optimal Control Governed by PDEs

- Gradient descent

Gateaux derivative

$$\begin{aligned} a_j &\leftarrow a_j - d\boxed{\frac{\mathrm{D}J}{\mathrm{D}a_j}}, \\ b_j &\leftarrow b_j - d\boxed{\frac{\mathrm{D}J}{\mathrm{D}b_j}}, \end{aligned} \qquad j = 1, \cdots, M.$$

Adjoint function

$$\begin{aligned} \frac{\mathrm{D}J}{\mathrm{D}a_i} &= \lambda_i a_i - \int_\Omega \sum_{m=1}^{M} \boxed{\varphi_m} \mathrm{inv}_i(\rho_m, O_m)\mathrm{d}\Omega, \\ \frac{\mathrm{D}J}{\mathrm{D}b_i} &= \mu_i b_i - \int_\Omega \sum_{m=1}^{M} \boxed{\phi_m} \mathrm{inv}_i(O_m, \rho_m)\mathrm{d}\Omega. \end{aligned} \qquad i = 1, \cdots, M.$$

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Adjoint Equations

The adjoint equation for φ_k is

$$\begin{cases} \dfrac{\partial \varphi_m}{\partial t} + \displaystyle\sum_{p\in\wp}(-1)^{|p|}\left(\sigma_{O;p}\varphi_m + \sigma_{\rho;p}\phi_m\right)_p = 0, & (\mathbf{x},t)\in Q,\\ \varphi_m = 0, & (\mathbf{x},t)\in\Gamma,\\ \varphi_m|_{t=1} = \tilde{O}_m - O_m(1), & \mathbf{x}\in\Omega. \end{cases}$$

Propagates backwards!

The adjoint equation for ϕ_k is

$$\begin{cases} \dfrac{\partial \phi_m}{\partial t} + \displaystyle\sum_{p\in\wp}(-1)^{|p|}\left(\tilde{\sigma}_{O;p}\varphi_m + \tilde{\sigma}_{\rho;p}\phi_m\right)_p = 0, & (\mathbf{x},t)\in Q,\\ \phi_m = 0, & (\mathbf{x},t)\in\Gamma,\\ \phi_m|_{t=1} = 0, & \mathbf{x}\in\Omega, \end{cases}$$

where

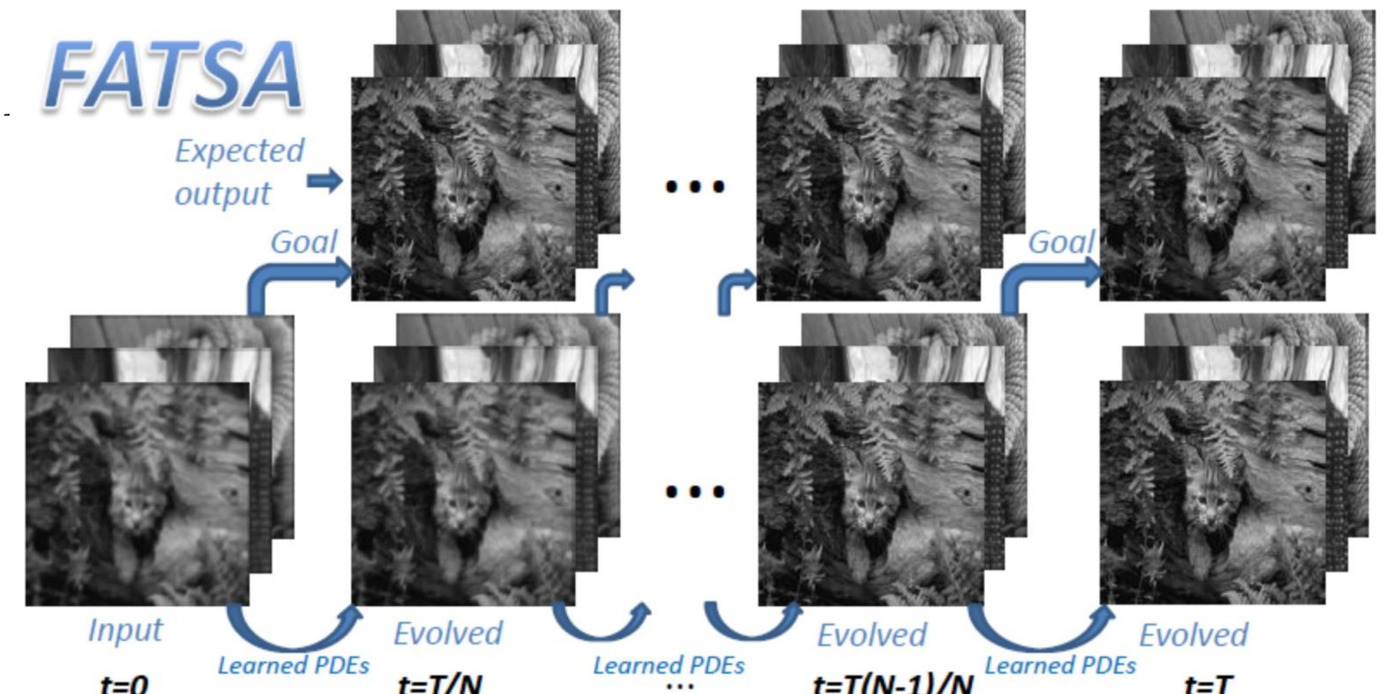
$\approx$ BP in NN

$$\tilde{\sigma}_{O;p} = \frac{\partial L_O}{\partial \rho_p} = \sum_{i=0}^{16} a_i \frac{\partial \mathrm{inv}_i(\rho, O)}{\partial \rho_p}, \quad \text{and} \quad \tilde{\sigma}_{\rho;p} = \frac{\partial L_\rho}{\partial \rho_p} = \sum_{i=0}^{16} b_i \frac{\partial \mathrm{inv}_i(O, \rho)}{\partial \rho_p}.$$

Layer-wise Optimization

- Minimize the difference from the ground truth at every time step.

$$\begin{cases} O_m^{n+1} = O_m^n + \Delta t \cdot \mathbf{inv}^T(O_m^n, \rho_m^n) \cdot \mathbf{a}^n, & n \geq 0, \\ \rho_m^n = \rho_m^{n-1} + \Delta t \cdot \mathbf{inv}^T(\rho_m^{n-1}, O_m^{n-1}) \cdot \mathbf{b}^{n-1}, & n \geq 1. \end{cases}$$



Zhao et al., *A Fast Alternating Time-Splitting Approach for Learning Partial Differential Equations*, Neurocomputing 2016.

So Complex …

- The above is our effort to set up the framework
- A user only have to prepare input/output training pair
- Once coefficients are computed, PDEs are obtained

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Experiments

- The same form of PDEs for different problems!

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Image Blur

RMSE=0.46, PSNR=54.88dB

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Image Blur

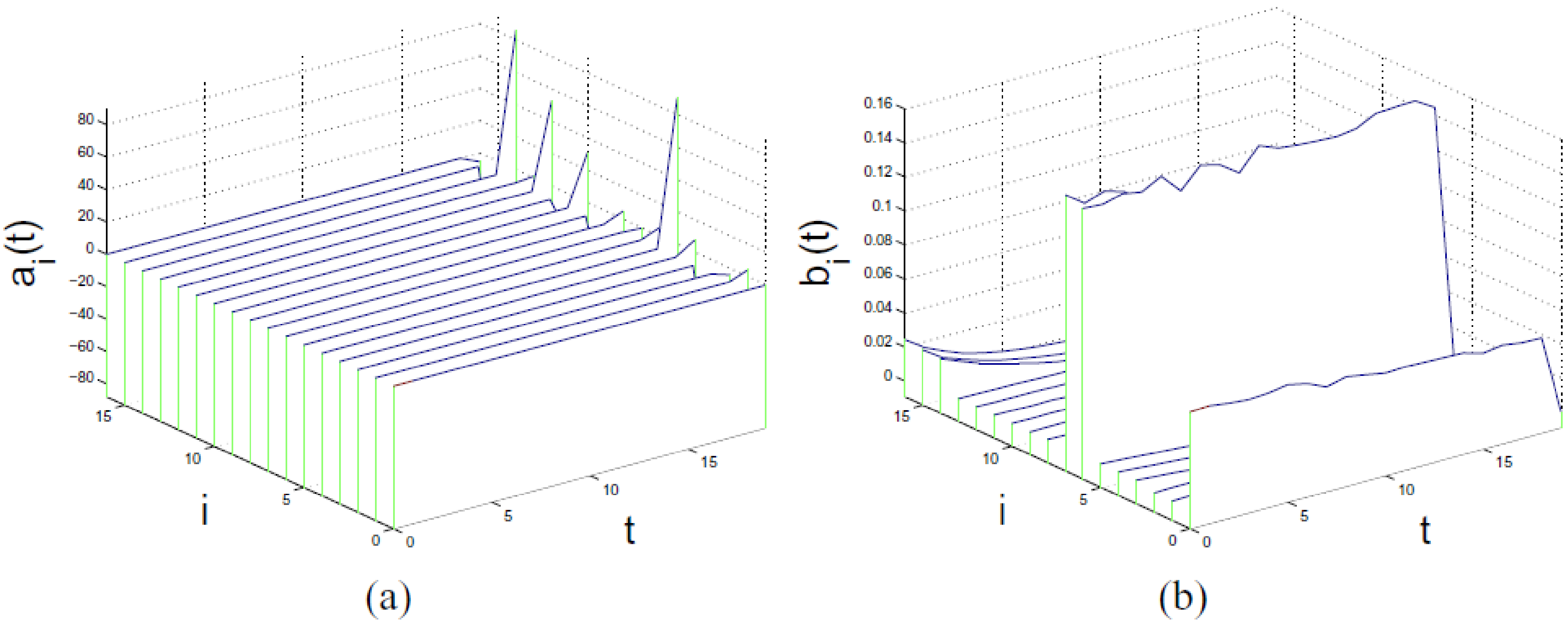
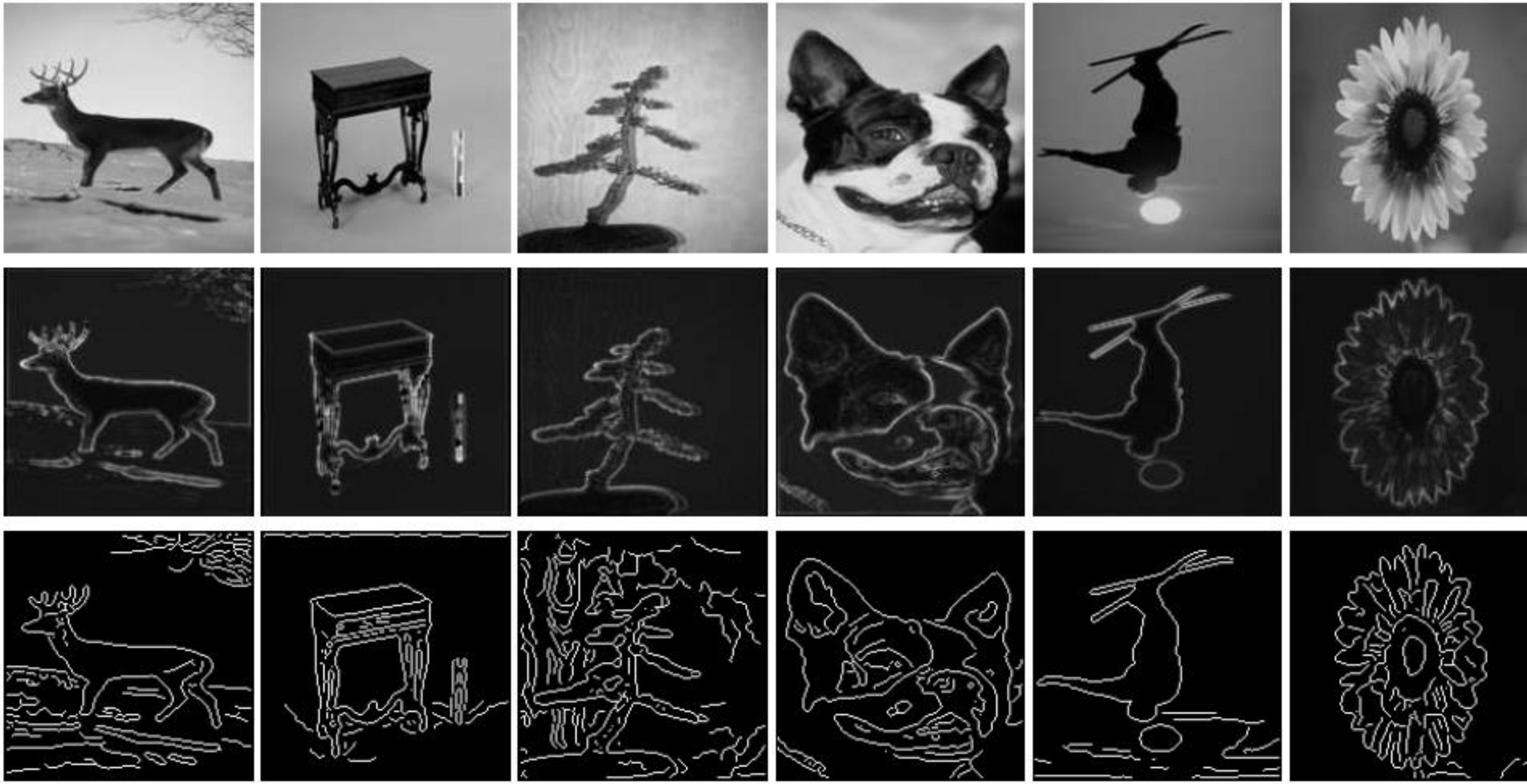


Figure 2: The learnt coefficients (a) a_i and (b) b_i, $i = 0, \cdots, 16$, for image blurring.

Standard heat equation: $a_7 = \text{const} > 0$, $a_i \equiv 0$, $i \neq 7$, and $b_j \equiv 0$, $j = 0, \cdots, 16$.

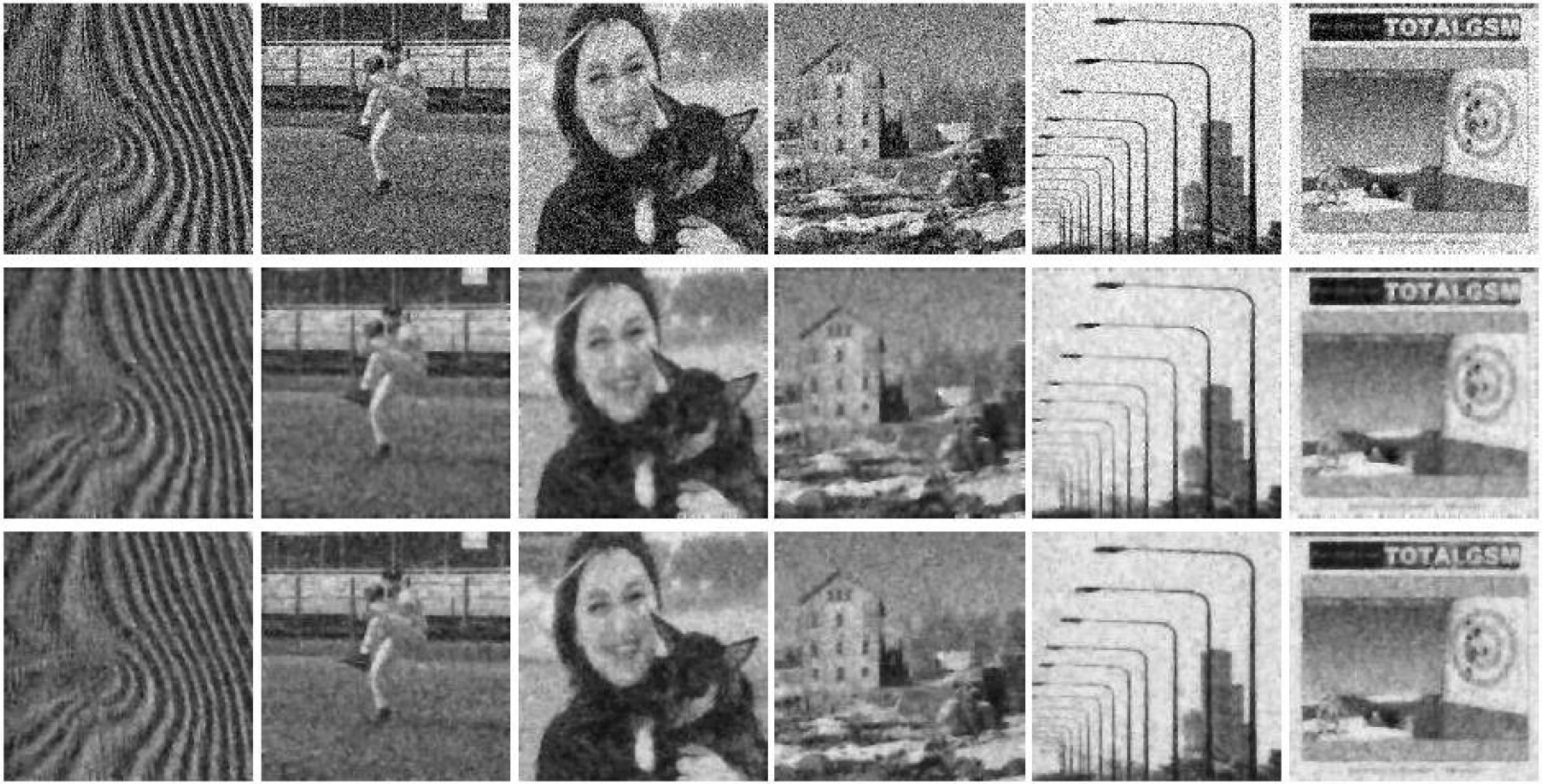
Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Perceptual Edge Detection



Liu, <u>Lin</u>, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

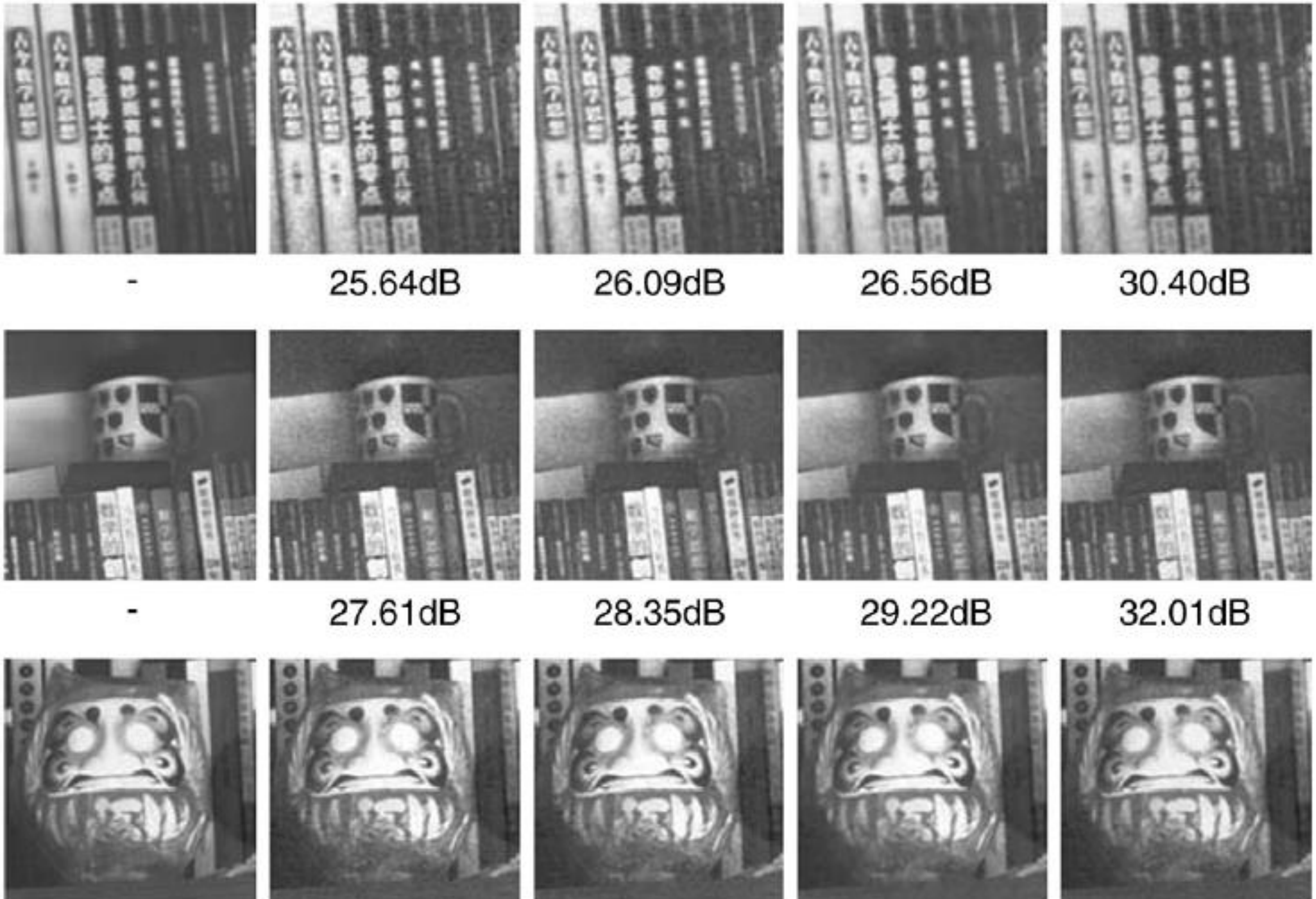
Image Denoising – Gaussian Noise



OUR: 27.87±2.07dB; Other PDE: 26.91±2.68dB

G. Gilboa, N. Sochen, and Y.Y. Zeevi. Image enhancement and denoising by complex diffusion processes. *IEEE TPAMI*, 26(8):1020–1036, 2004.

Image Denoising – Real Noise



Noiseless	Noisy	ROF	TV-l_1	LPDE
-	25.64dB	26.09dB	26.56dB	30.40dB
-	27.61dB	28.35dB	29.22dB	32.01dB
-	27.98dB	28.67dB	29.52dB	32.29dB

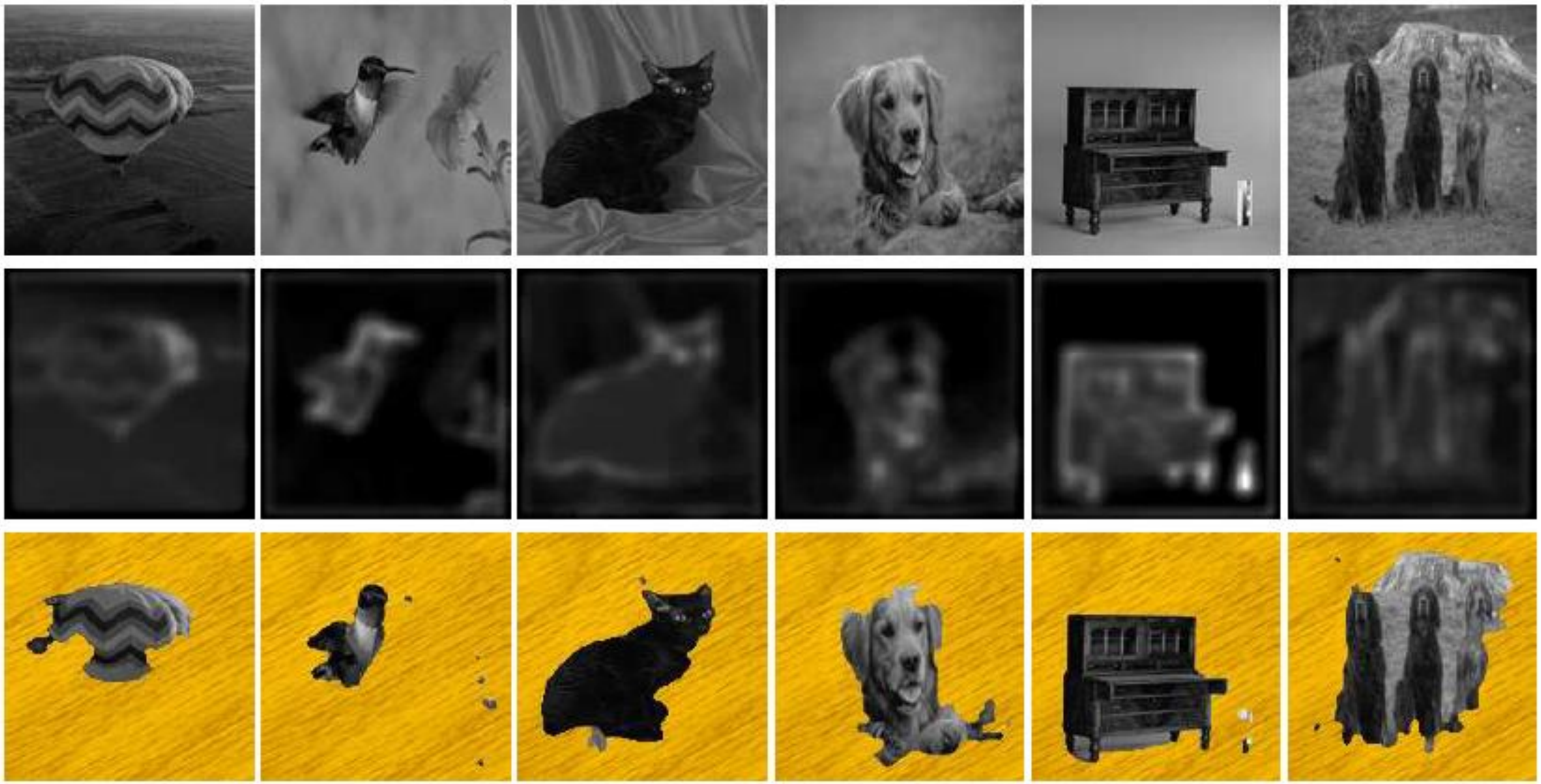
Plane Detection

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Plane Detection

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Plane Detection

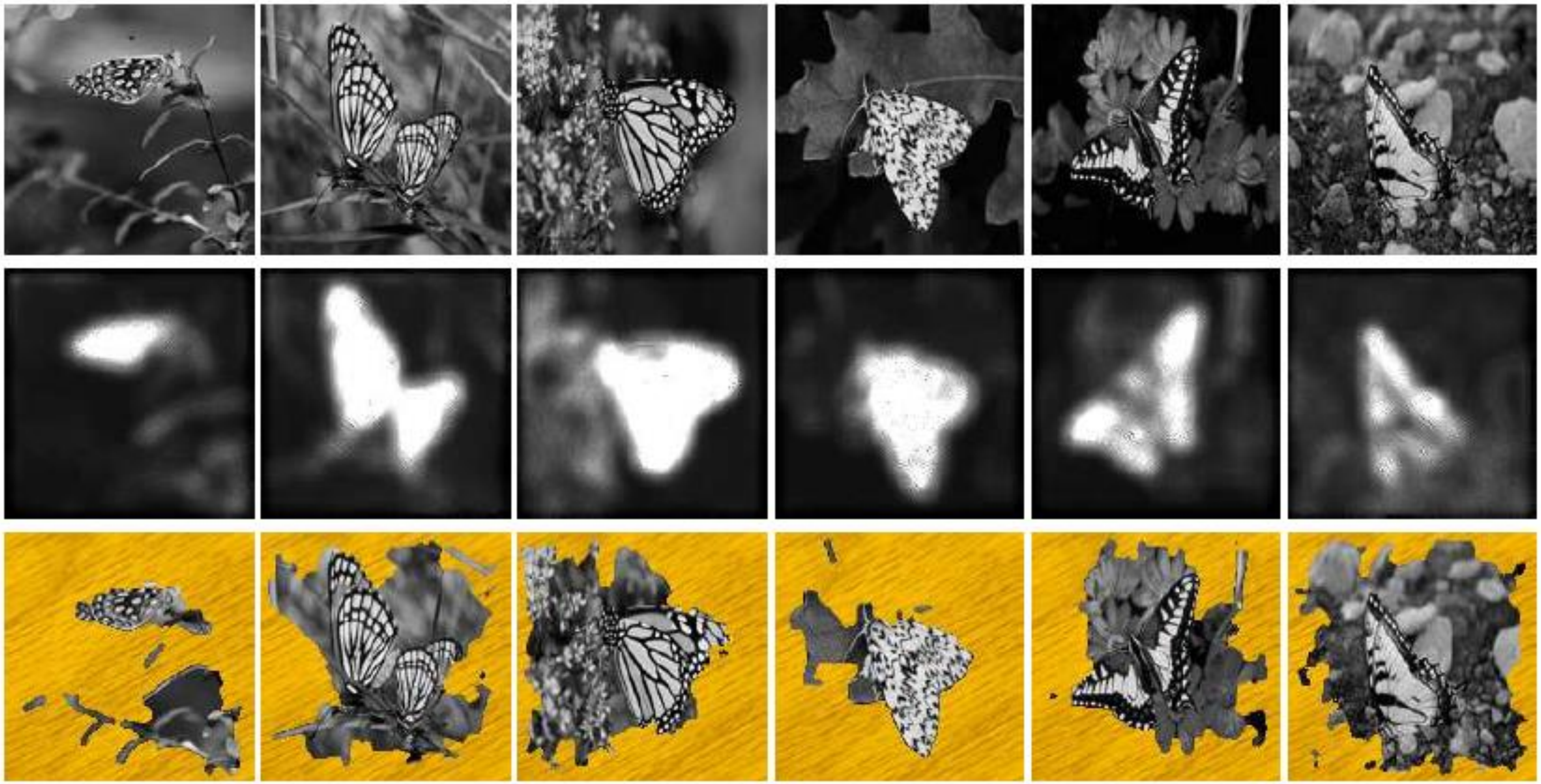


Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Butterfly Detection

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Butterfly Detection



Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Butterfly Detection

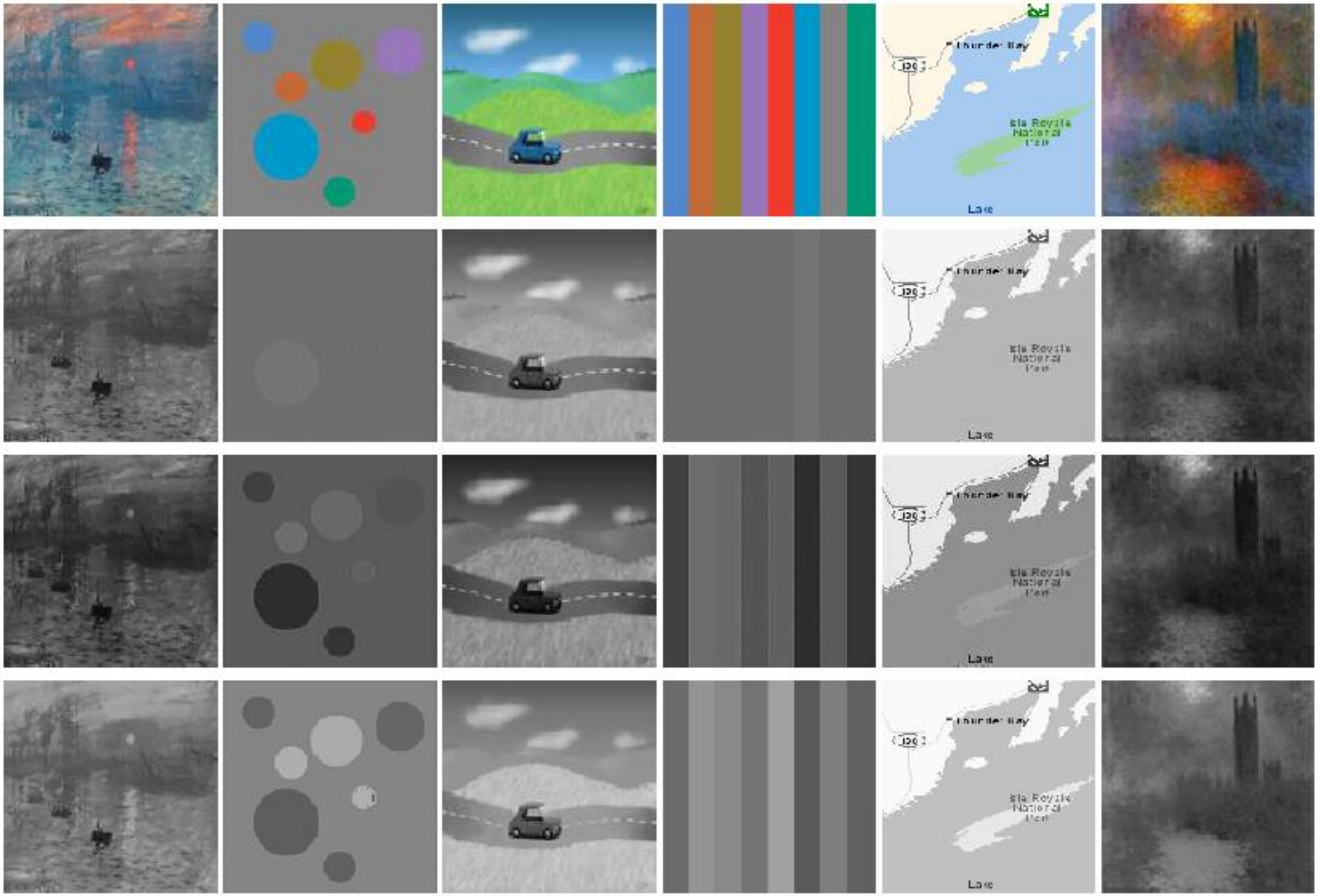
Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Handling Color Images

- Correlation among colors is tricky!
- Multi-channels + one indicator function
- 69 fundamental differential invariants

Liu, Lin, Zhang, Tang, and Su, *Toward Designing Intelligent PDEs for Computer Vision: A Data-Based Optimal Control Approach*, Image and Vision Computing, 2013.

Experiments - Color2Gray



Experiments - Demosaicking

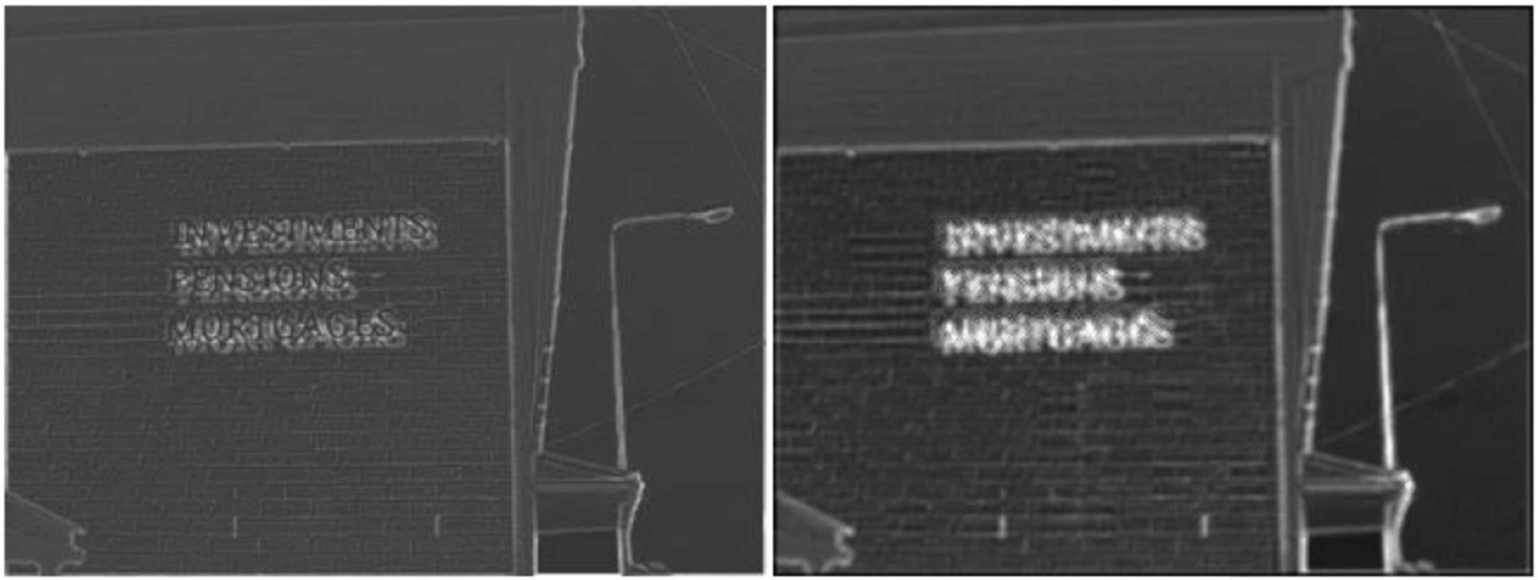
Full image Zoomed region BI SA [18] DFAPD [20] BI + 13 layers

Experiments – Text Detection

(a) Input image (b) Output image

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

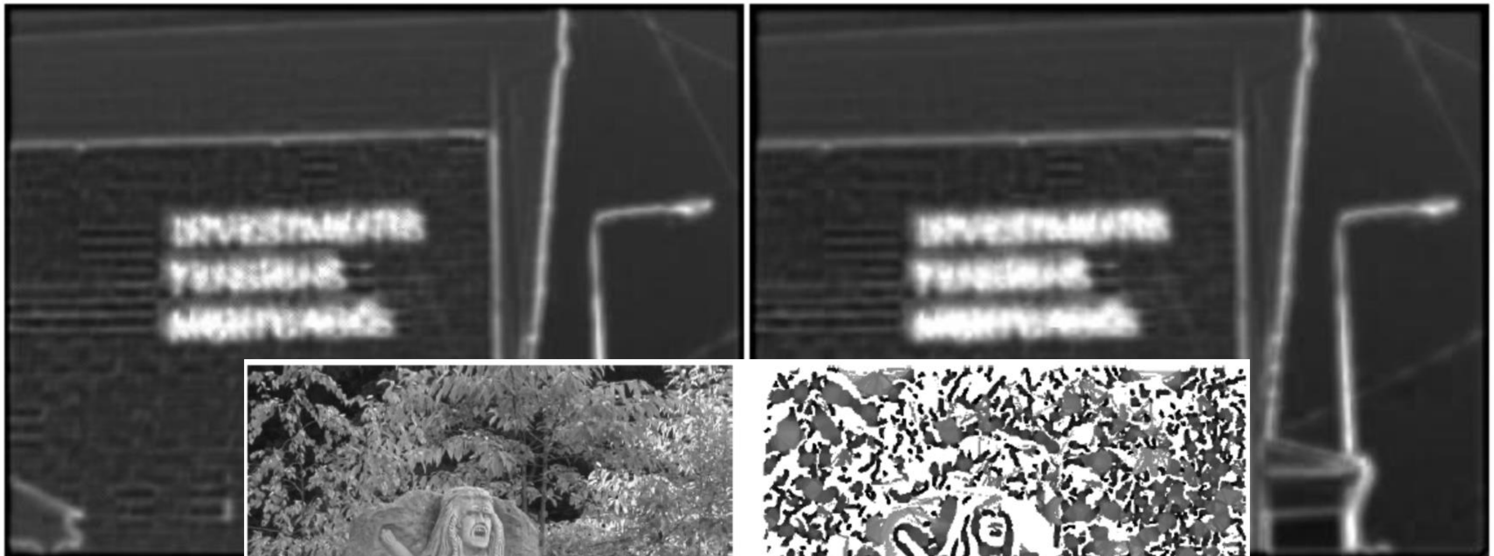
Experiments – Text Detection

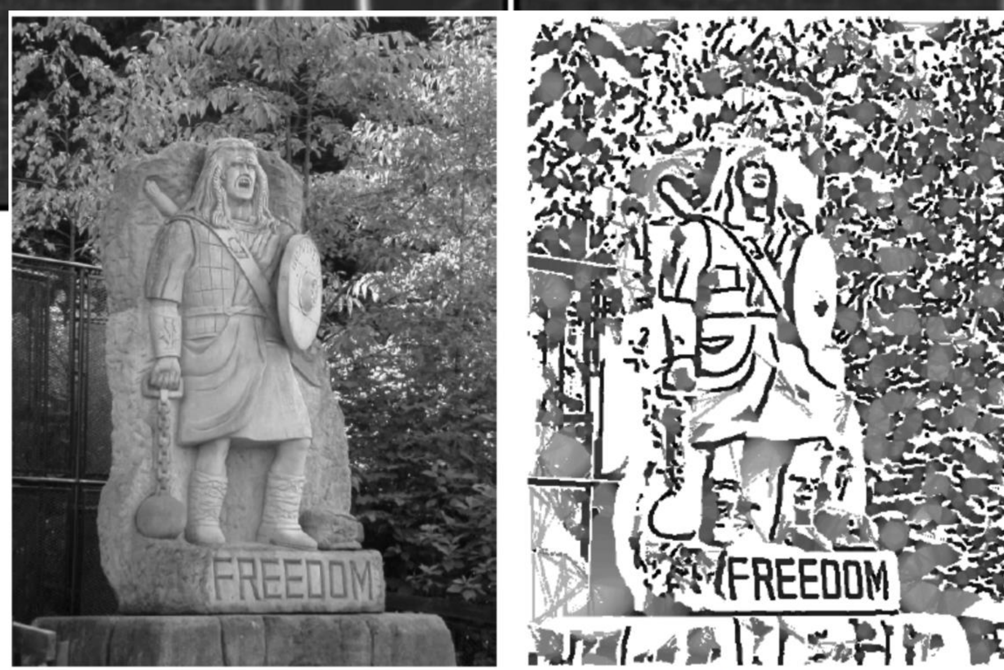


(c) $n = 1$ (d) $n = 3$

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection





(a) Input (b) SWT

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection

ICDAR

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection

SVT

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection

Table 2: Comparison with most recent text detection results on the ICDAR 2005 test database. The results of other methods are quoted from [12, 26].

Methods	Description	Precision	Recall	F-measure
Our method	**Hybrid**	**0.87**	**0.67**	**0.76**
Pan et al. [12]	Hybrid	0.67	0.70	0.69
Huang et al. [26]	CC (SWT)	0.81	0.74	0.72
Yao et al. [25]	CC (SWT)	0.69	0.66	0.67
Epshtein et al. [5]	CC (SWT)	0.73	0.60	0.66
Chen et al. [47]	CC (MSER)	0.73	0.60	0.66
Neumann and Matas [49]	CC (MSER)	0.65	0.64	0.63
Wang et al. [50]	CC	0.77	0.61	0.68
Yi and Tian [51]	CC	0.71	0.62	0.63
Zhang and Kasturi [52]	CC	0.73	0.62	-
Yi and Tian [21]	CC	0.71	0.62	0.62
Lee et al. [40]	Sliding window	0.66	0.75	0.70

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection

Table 3: Comparison with most recent text detection results on the ICDAR 2011 test database. These results are from the papers [24, 48].

Methods	Description	Precision	Recall	F-measure
Our method	**Hybrid**	**0.88**	**0.69**	**0.78**
Yin et al. [24]	CC (MSER)	0.86	0.68	0.76
Neumann and Matas [48]	CC (MSER)	0.85	0.68	0.75
Ye et al. [4]	CC (MSER)	0.89	0.62	0.73
Neumann and Matas [23]	CC (MSER)	0.79	0.66	0.72
Shi et al. [53]	CC (MSER)	0.83	0.63	0.72
Koo et al. [54]	CC (MSER)	0.83	0.63	0.71
Huang et al. [26]	CC (SWT)	0.82	0.75	0.73
Yi and Tian [51]	CC	0.81	0.72	0.71
Wang et al. [50]	CC	0.71	0.57	0.63

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection

Table 4: Comparison with most recent text detection results on the SVT 2010 database. The results of other methods are quoted from respective papers.

Methods	Description	Precision	Recall	F-measure
Our method, trained on the ICDAR 2011 database	**Hybrid**	**0.72**	**0.41**	**0.52**
Yin et al. [24], trained on the SVT 2011 database trained on the ICDAR 2011 database	CC (MSER)	 0.66 0.62	 0.41 0.32	 0.51 0.42
Phan et al. [55]	CC	0.50	0.51	0.51
Epshtein et al. [5]	CC (SWT)	0.54	0.42	0.47

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Experiments – Text Detection

Table 5: Comparison with most recent text detection results on the SVT 2011 database. The results of other methods are quoted from respective papers.

Methods	Description	Precision	Recall	F-measure
Our method, trained on the ICDAR 2011 database	**Hybrid**	**0.65**	**0.39**	**0.49**
Wang et al. [6]	CC	0.67	0.29	0.41
Neumann et al. [38]	CC (MSER)	0.19	0.33	-

Zhenyu Zhao, Cong Fang, Zhouchen Lin, and Yi Wu, *A Robust Hybrid Method for Text Detection in Natural Scenes by Learning-based Partial Differential Equations*, Neurocomputing, 2015.

Grand Picture

- What are the PDEs that govern visual processing?

There should be!

Grand Picture

- How to find the PDEs?

Symmetries or Invariances

- Newton Laws: Galilean Transformation
- Maxwell Equations: Lorentzian Transformation
- Special Relativity: FitzGerald–Lorentz–Einstein Transformation
- General Relativity: Gauge Invariance
- String Theory: Super-Symmetry
- Higgs Particle: Local Gauge Invariance of Young-Mills Equation

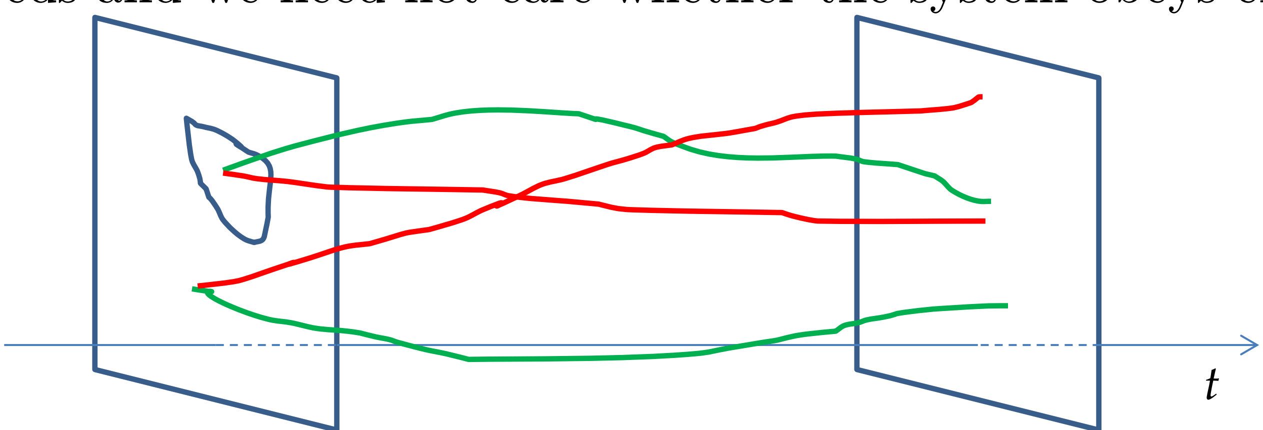
Grand Picture

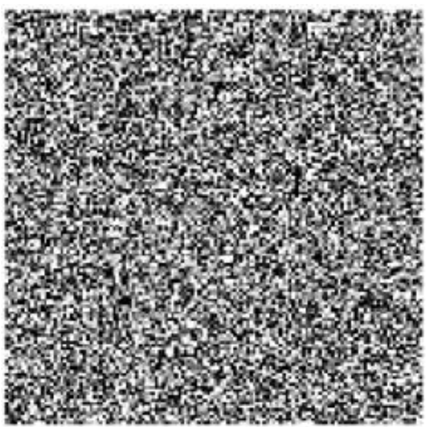
- Learning based PDEs only requires that its output is close to that of real visual system when the input is a meaningful image.

For example, although

$$O_1(\mathbf{x},t) = \|\mathbf{x}\|^2 \sin t \text{ and } O_2(\mathbf{x},t) = (\|\mathbf{x}\|^2 + (1-t)\|\mathbf{x}\|)(\sin t + t(1-t)\|\mathbf{x}\|^3)$$

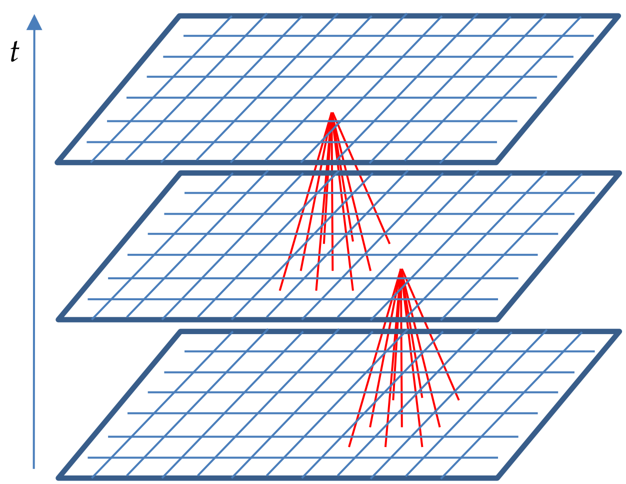
are very different functions, they initiate from the same function at $t=0$ and also settle down at the same function at time $t=1$. So both functions fit our needs and we need not care whether the system obeys either function.

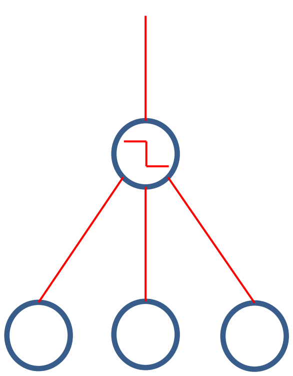




Conclusions and Future Work

- LPDEs is a promising framework to solve different computer vision and image processing problems in a unified way.
- More invariants and more complex combinations are yet to be explored.
- Biological explanation of the LPDEs is also interesting.
- Connections to deep learning?





Thanks!

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- http://www.cis.pku.edu.cn/faculty/vision/zlin/zlin.htm